$$\lambda = \frac{\max \text{Prob}(\text{data} \mid H_0)}{\max \text{Prob}(\text{data} \mid H_1)} =$$

H_0: $\text{Prob}(\text{data} \mid H_0) = (2\pi\sigma^2)^{-\frac{1}{2}\sum_{j=1}^{k} n_j} e^{-\frac{1}{2\sigma^2}\sum_{j=1}^{k}\sum_{i=1}^{n_j}(y_{ji}-\mu)^2}$ is

maximized at $\mu = \hat{\mu}_{MLE} = \bar{\bar{y}} = \sum_j \sum_i y_{ji} / \sum_j n_j$

and $\sigma^2 = \hat{\sigma}^2_{MLE,0} = \sum_j \sum_i (y_{ji} - \bar{\bar{y}})^2 / \sum_j n_j$

to be $\max \text{Prob}(\text{data} \mid H_0) = (2\pi \hat{\sigma}_{MLE,0})^{-\frac{1}{2}\sum_j n_j} \cdot e^{-\frac{1}{2}\sum_j n_j}$

H_1: $\text{Prob}(\sim \mid H_1) = \sim\ \sim e^{\sim \Sigma\Sigma (y_{ji} - \mu_j)^2}$ is

maximized at $\mu_j = \hat{\mu}_{MLE,j} = \bar{y}_j = \sum_i y_{ji} / n_j$

and $\sigma^2 = \hat{\sigma}^2_{MLE,1} = \sum_j \sum_i (y_{ji} - \bar{y}_j)^2 / \sum_j n_j$

to be $\max \text{Prob}(\text{data} \mid H_1) = (2\pi \hat{\sigma}_{MLE,1})^{\sim} \cdot e^{-\!-}$.

Therefore, $\lambda = \left(\frac{\hat{\sigma}_{MLE,1}}{\hat{\sigma}_{MLE,0}}\right)^{\frac{1}{2}\sum_j n_j} = \left(\frac{\sum_j \sum_i (y_{ji} - \bar{y}_j)^2}{\sum_j \sum_i (y_{ji} - \bar{\bar{y}})^2}\right)^{\frac{1}{2}\sum_j n_j}$

Define $N := \sum_j n_j$.

$$= \left[\frac{\sum_j \sum_i (y_{ji} - \bar{y}_j)^2}{\sum_j \sum_i (y_{ji} - \bar{y}_j)^2 + \sum_j n_j (\bar{y}_j - \bar{\bar{y}})^2}\right]^{\frac{N}{2}}$$

by splitting sums (I don't know how)

$$= \left[\frac{1}{1 + \frac{\sum_j n_j (\bar{y}_j - \bar{\bar{y}})^2}{\sum_j \sum_i (y_{ji} - \bar{y}_j)^2}}\right]^{\frac{N}{2}}$$

by dividing bothsides by numerator

$$= \left[\frac{1}{1 + \frac{\frac{k-1}{k-1}\sum_j n_j (\bar{y}_j - \bar{\bar{y}})^2}{\frac{N-k}{N-k}\sum_j \sum_i (y_{ji} - \bar{y}_j)^2}}\right]^{\frac{N}{2}}$$

by inventing new fractions

$$= \left[\frac{1}{1 + \frac{k-1}{N-k} F}\right]^{\frac{N}{2}}$$

by recognizing $F \sim F(\cancel{N-1}^{\,k-1}, \cancel{N-1}^{\,N-k})$.

We reject H_0 if F is sufficiently large. How large? F-chart.